Initial & Final State Effects in the melting Color Glass Condensate

Raju Venugopalan

BROOKHAVEN NATIONAL LABORATORY

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Introduction:

- Analytical & numerical studies of initial & final state effects in high energy hadronic scattering.

- Is "k_t factorization" of quark and gluon of gluon and quark cross−sections a good assumption in p/d−A & A−A collisions?

- Address relative importance of multiple scattering "Cronin" vs quantum evolution (geometrical scaling) effects on gluon and quark production in p/d−A and A−A collisions

- How does one systematically treat the evolution of soft and hard partonic modes in the final state of heavy ion collisions? Does the system thermalize?

Can address all of these issues in the CGC framework.

This talk is based on the following work:

a) High p_t quark production in A−A collisions
(F. Gelis & RV, hep−ph/0310090−PRD, in press)

b) k_t factorization in gluon & quark production in pA collisions (J.−P. Blaizot, F. Gelis & RV, in preparation)

c) Numerical studies of gluon production in A−A collisions [A. Krasnitz, Y. Nara, RV, NPA 717, 268, (2003); J. Jalilian−Marian, Y. Nara, RV, Phys. Lett. B577, 54 (2003)]

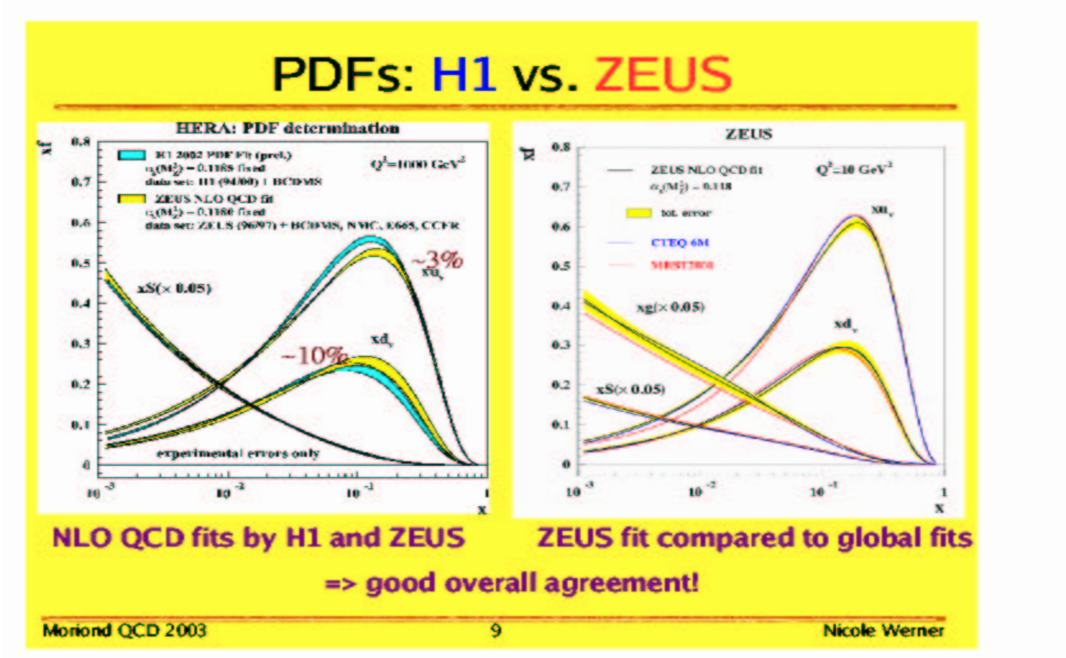
d) Dynamical evolution of particle & field modes in ϕ^4 theory with CGC initial conditions [work in progress with S. Jeon, L. McLerran, S. Weinstock]

Outline:

- The Color Glass Condensate
- Gluon & Quark production to lowest order in the sources (the dilute/pp case)
- Gluon & Quark production to lowest order in one source & all orders in the other (semi–dense/pA case)
- Gluon & Quark production to all orders in both sources (dense/AA case)
- Dynamical evolution of soft & hard modes at late times in AA collisions–a toy ϕ^4 –model

The Color Glass Condensate

review: E. Iancu, RV, hep−ph/0303204



High Energy Nuclear Wave−function

- *Small x partons−large occupation #−described by classical color field* A_μ^a
- *Large x partons−static color charges* ρ^a
- *Classical field of the nucleus obeys the Yang−Mills eqns:*

$$[D_\mu, F^{\mu\nu}]^a = \delta^{\nu+}\delta(x^-)\,\rho^a(x_\perp)$$

- *Color sources* ρ^a *are random and are described by the distribution functional* $W_{x_0}[\rho]$ *where x_0 separates "fields" and "sources"*

- *Observables are calculated in the classical field– for fixed* ρ^a *and then averaged over* ρ^a *with* $W_{x_0}[\rho]$ *to obtain the gauge–invariant expectation value:*

$$O = \int [d\rho_a]\, W_{x_0}[\rho_a]\, \mathcal{O}[\rho_a]$$

- *For large nuclei, without quantum evolution,*

$$W_{x_0}[\rho_a] = \exp\left(- \int d^2 x_\perp \frac{\rho^a \rho^a}{2\Lambda_s^2} \right)$$

where $Q_s \approx \Lambda_s$ *is the saturation scale*

- $W_{x_0}[\rho]$ *evolves with decreasing* x_0 *–obeys the non–linear RG eqn:*

$$\frac{\partial W_{x_0}[\rho]}{\partial \ln(1/x_0)} = \frac{1}{2} \int_{x_\perp, y_\perp} \frac{\partial}{\partial \rho_a(x_\perp)} \chi_{ab}(x_\perp, y_\perp) \frac{\partial}{\partial \rho_b(y_\perp)} W_{x_0}[\rho]$$

  - *The kernel* χ_{ab} *contains all orders in* ρ^a
  - *Reduces to the BFKL kernel for low densities.*

JIMWLK –Jalilian–Marian,Iancu,McLerran,Weigert,Leonidov,Kovner

Gluon & Quark production in the dilute/pp region

($\rho^{1,2}/k_t^2 << 1$)

- Collinear factorization: *Incoming partons have k_t =0. Applicable for* $Q \sim \sqrt{s} >> \Lambda_{QCD}$

  –Quark & Gluon Dists. evaluated at the scale Q^2

  –are universal

- k_t factorization: *Incoming partons have k_t*

  –applicable when $Q, \sqrt{s} >> \Lambda_{QCD}$; $Q << \sqrt{s}$

  –described by unintegrated parton dists. $\phi_{p/A}(k_\perp)$

  Collins, Ellis; Catani, Ciafaloni & Hautmann

  Is this k_t scale the saturation scale? k_t~ Q_s ?

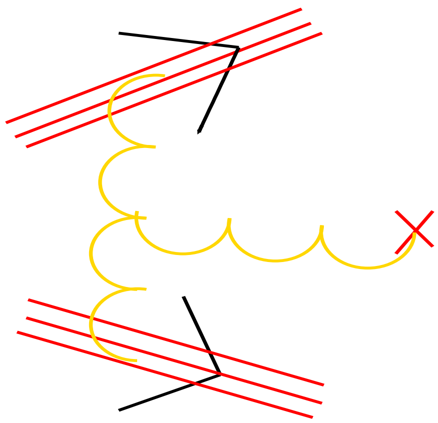
  Levin,Ryskin,Shabelski,Shuvaev

  Several phenomenological studies by LRSS & Hagler et al. studying spectra & correlations in pp–collisions

  (related approach by Raufeisen,Kopeliovich,Tarasov)

- *The CGC is a powerful formalism to study these issues . Both Collinear factorization and k_t factorizations arise as specific limits in this formalism.*

- Inclusive gluon production in hadronic collisions to lowest order in ρ^1, ρ^2 and in α_S expressed in k_t factorized form



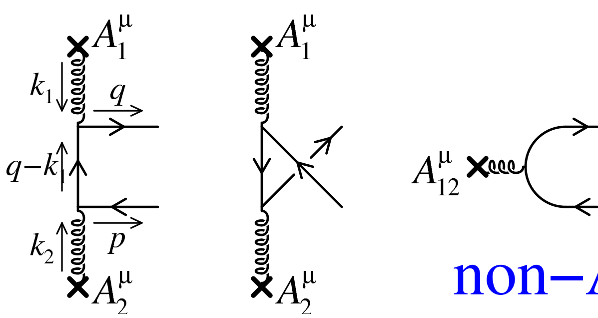
Kovner,McLerran,Weigert
Kovchegov, Rischke
Gyulassy, McLerran

This diagram in $A^\tau = 0$ gauge is equivalent to sum of all bremsstrahlung diagrams in covariant gauge

- **Inclusive pair−production in CGC framework**

Gelis, RV

Work in $\partial_\mu A^\mu = 0$ gauge



Abelian

$A_{12}^\mu \propto 0(\rho 1 \rho 2)$

non−Abelian−vertex here is the Lipatov vertex C^μ

- $$\frac{\partial\sigma}{dy_p dy_q d^2p_\perp d^2q_\perp} = \frac{1}{(2\pi)^6}\frac{1}{(N_c^2-1)^2}\int\frac{d^2k_{1\perp}}{(2\pi)^2}\frac{d^2k_{2\perp}}{(2\pi)^2}\delta(\vec{k}_{1\perp}+\vec{k}_{2\perp}-\vec{p}_\perp-\vec{q}_\perp)$$
$$\phi_1(k_{1\perp})\phi_2(k_{2\perp})\frac{\mathrm{Tr}\left(|m_{ab}^{-+}(k_1,k_2;q,p)|^2\right)}{k_{1\perp}^2 k_{2\perp}^2}$$

$|m_{ab}^{-+}(k_1,k_2;q,p)|^2$ is identical to Collins & Ellis's k_t factorization result.

$$\frac{d\phi_1(k_{1\perp},x_\perp)}{d^2x_\perp} = \frac{\pi g^2}{k_\perp^2}\int d^2r_\perp e^{-i\vec{k}_\perp\cdot\vec{r}_\perp}\langle\rho_a(x_\perp+\frac{r_\perp}{2})\rho_a(x_\perp-\frac{r_\perp}{2})\rangle_\rho$$

is the unintegrated gluon distribution $= \dfrac{\pi g^2(N_c^2-1)\mu^2}{k_\perp^2}$
in the Gaussian MV–model

- $\dfrac{\mathrm{Tr}\left(|m_{ab}^{-+}(k_1,k_2;q,p)|^2\right)}{k_{1\perp}^2 k_{2\perp}^2}$ is well defined in in the collinear limit of $|k_{1\perp}|, |k_{2\perp}| \to 0$

$|M|^2_{gg\to q\bar{q}}$ after integration over azimuthal angles

Recover lowest order collinear factorization result!

Gluon & Quark production in the semi–dense/pA region

$$\left(\rho_1/k_t^2 << 1, \rho_2/k_t^2 \sim 1\right)$$

Blaizot, Gelis, RV

- Solve classical Yang–Mills eqns.

$$[D_\mu, F^{\mu\nu}] = J^\nu \; ; \; [D_\nu, J^\nu] = 0$$

with two light cone sources

$$J^{\nu,a} = \underbrace{\delta^{\nu+}\delta(x^-)\rho_1^a(x_\perp)}_{\textit{proton source}} + \underbrace{\delta^{\nu-}\delta(x^+)\rho_2^a(x_\perp)}_{\textit{nuclear source}}$$

- $\boxed{\partial_\mu A^\mu = 0}$ => equations can be written as

$$(2\partial^+\partial^- - \nabla_\perp^2)A^\nu = J^\nu + ig\,[A_\mu, F^{\mu\nu} + \partial^\mu A^\nu]$$

need $A^\mu_{1\infty}$ =order $O(\rho_1)$ in proton & order $O(\rho_2^n)\,;\, n\to\infty$ in nucleus

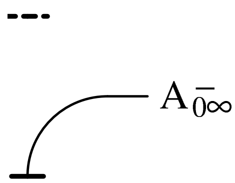
$$\begin{aligned}
(\partial^- + igA^-_{0\infty}\cdot T)J^+_{1\infty} &= 0 \\
(2\partial^+\partial^- - \nabla_\perp^2 + igA^-_{0\infty}\cdot T\partial^+)A^+_{1\infty} &= J^+_{1\infty} \\
(2\partial^+\partial^- - \nabla_\perp^2 + 2igA)0\infty^- \cdot T\partial^+)A^i_{1\infty} &= ig(A^-_{0\infty}\cdot T)\partial^i A^+_{1\infty} - ig(\partial^i A^-_{0\infty}\cdot T)A^+_{1\infty} \\
A^-_{1\infty} &= \frac{1}{\partial^+}(\partial^i A^i_{1\infty} + \partial^- A^+_{1\infty})
\end{aligned}$$

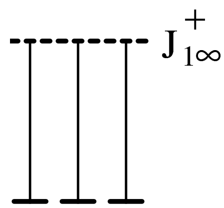
$$A^-_{0\infty} = -\delta(x^+)\frac{1}{\nabla_\perp^2}\rho_2(x_\perp) \qquad J^+_{1\infty} \to A^+_{1\infty} \to A^i_{1\infty} \to A^-_{1\infty}$$

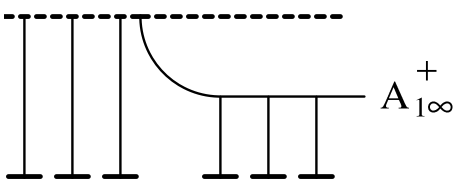
Diagrammatic Representation

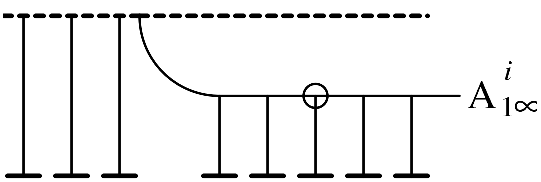
Solid line = 1 power of ρ_2

Dashed line=1 power of ρ_1



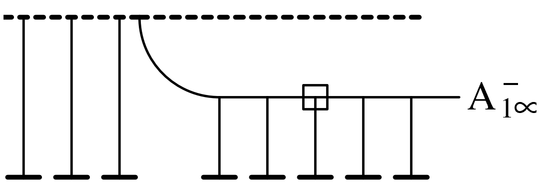


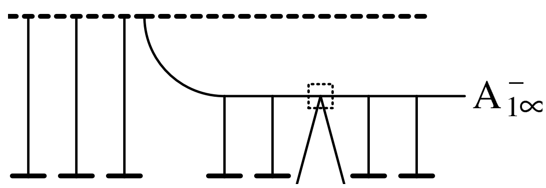


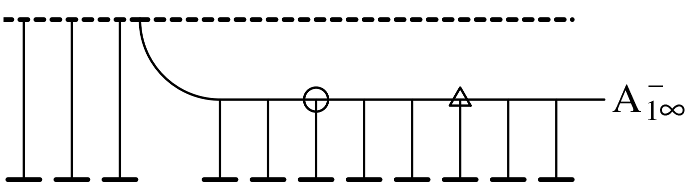


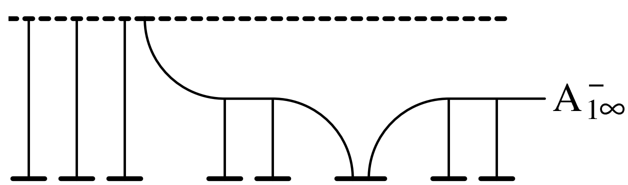
Circular vertex is the vertex

Γ^{i-+} for the transition $A^{+}_{1\infty} \to A^{i}_{1\infty}$









- The field $A^{-}_{1\infty}$ can be computed from the gauge condition $\partial_\mu A^\mu = 0$

- The gluon field produced in pA has the compact form

$$q^2 \tilde{A}^{\mu}_{1\infty}(q) = i \int \frac{d^4 k}{(2\pi)^4} \left(C^{\mu}_U \tilde{U}(k_2) + C^{\mu}_V \tilde{V}^{\mu}(k_2) + C^{\mu}_1 \tilde{1}(k_2) \right) 2\pi\delta(k^-) \frac{\rho_1(k_\perp)}{k^2}$$

with $k_2 = q - k_1$ & U and V are path ordered Wilson lines containing all orders in the nuclear source ρ_2

$\tilde{1}(k_2)$ is a 3–D delta–fn in momentum space

- The usual Lipatov vertex is simply $C^{\mu}_L = C^{\mu}_U + \frac{1}{2} C^{\mu}_V$

  For gluons produced on shell (q^2=0), one finds remarkably: $C^{\mu}_1 = 0;\ C_U \cdot C_V = C^2_V = 0;\ and\ C^2_U = C^2_L = -\frac{4 k_{1\perp}^2 k_{2\perp}^2}{q^2_\perp}$

- Only bi–linears of the Wilson line U survive in the squared amplitude! $\sum_\lambda |q^2 \tilde{A}^{\mu}_{1\infty} \varepsilon^{(\lambda)}_\mu(q)|^2$

- Final result for the gluon multiplicity in pA

$$\boxed{N_g = \frac{4g^2 N_c}{\pi^2 (N_c^2 - 1) q_\perp^2} \int \frac{d^3 q}{(2\pi)^3 2E_q} \frac{d^2 k_\perp}{(2\pi)^2} \int d^2 x_\perp \frac{d\phi_p(k_\perp, x_\perp)}{d^2 X_\perp} \frac{d\phi_A(q_\perp - k_\perp, x_\perp - b)}{d^2 X_\perp}}$$

k_t factorized into product of proton * nuclear unintegrated distributions

Kovchegov, Mueller
Kovchegov,Tuchin
Kovchegov,Kharzeev,Tuchin

$\phi_A(k_t, x_\perp) \propto < U^\dagger_{ab} U_{bc} >_{\rho^2}$ –is non–linear–contains gluon density to all orders–proportional to gluon density at large k_t

- Exactly equivalent to result of Dumitru & Mclerran in $A^\tau = 0$ gauge

Dumitru,Jalilian–Marian,Gelis

- Cronin effect?

Cronin Effect in Gluon Production

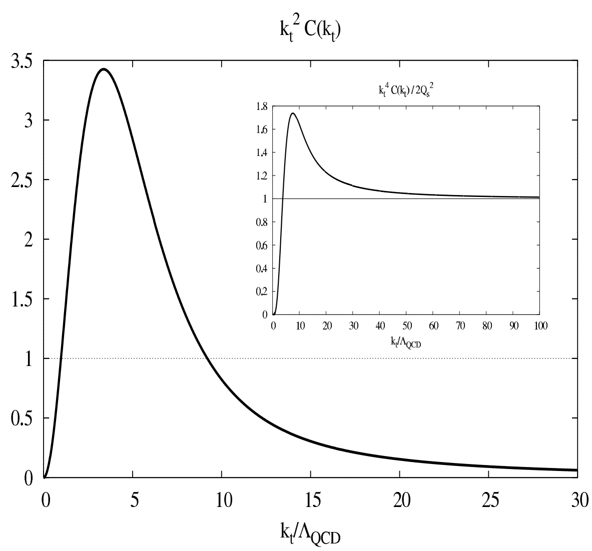
Consider Gaussian random sources–corresponds exactly to Glauber scattering of proj. parton on target

Accardi

$$W[\rho_2] = \exp\left(-\int d^2x_\perp \frac{\rho_{2a}(x_\perp)\rho_{2a}(x_\perp)}{2\mu^2}\right)$$

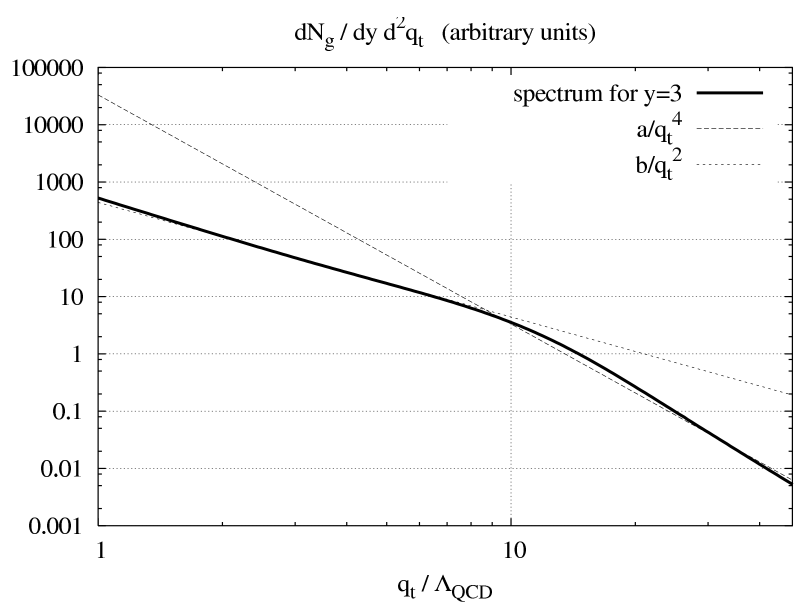
Expression for gluon multiplicity simplifies to

$$\frac{dN}{d^2q_\perp dy} = \frac{1}{16\pi^3 q_\perp^2}\int \frac{d^2k_\perp}{(2\pi)^2}(q_\perp - k_\perp)^2 C(q_\perp - k_\perp)\,\phi_p(k_\perp)$$



$$k_\perp^2 C(k_\perp) \approx \frac{g^2 N_c \mu^2}{k_\perp^2}$$ for large k_t

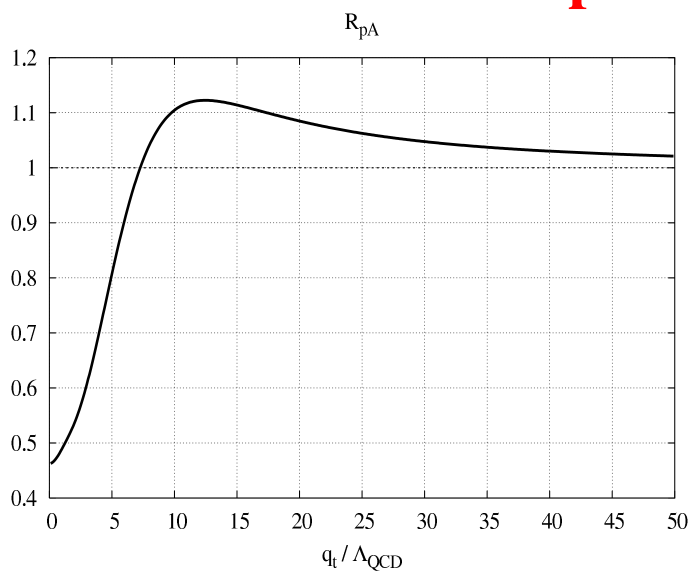
Gelis & Peshier



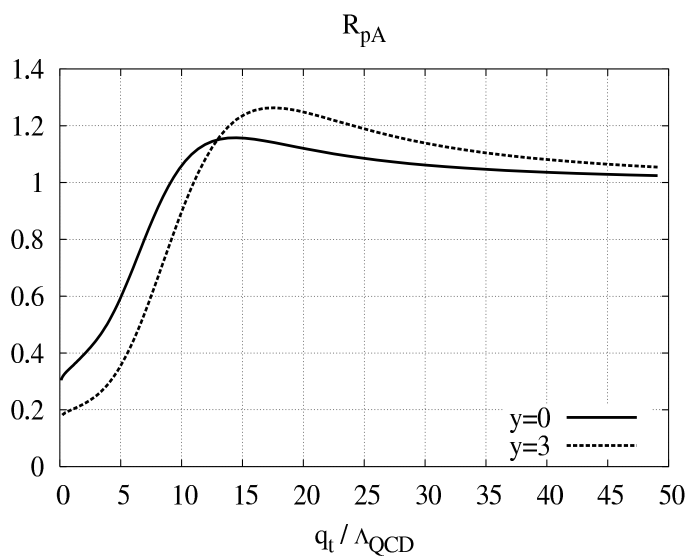
Explicitly compute spectrum predicted by Dumitru&McLerran

phenomenology by Lenaghan&Tuominen

Compute R_pA

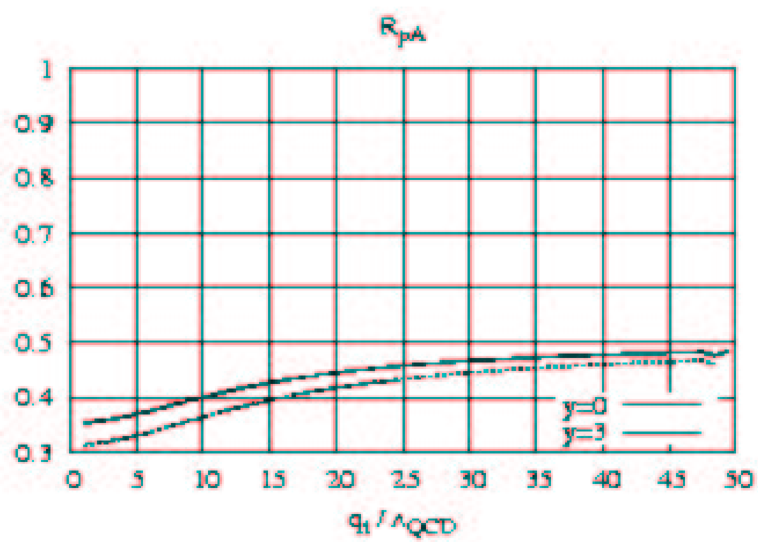


MV–model for fixed Q_s



MV–model with naive
quantum evolution
a la Golec–Biernat–Wusthoff

$$Q_s^A = A^{1/3}\left(\frac{x_0}{x}\right)^{\lambda}$$

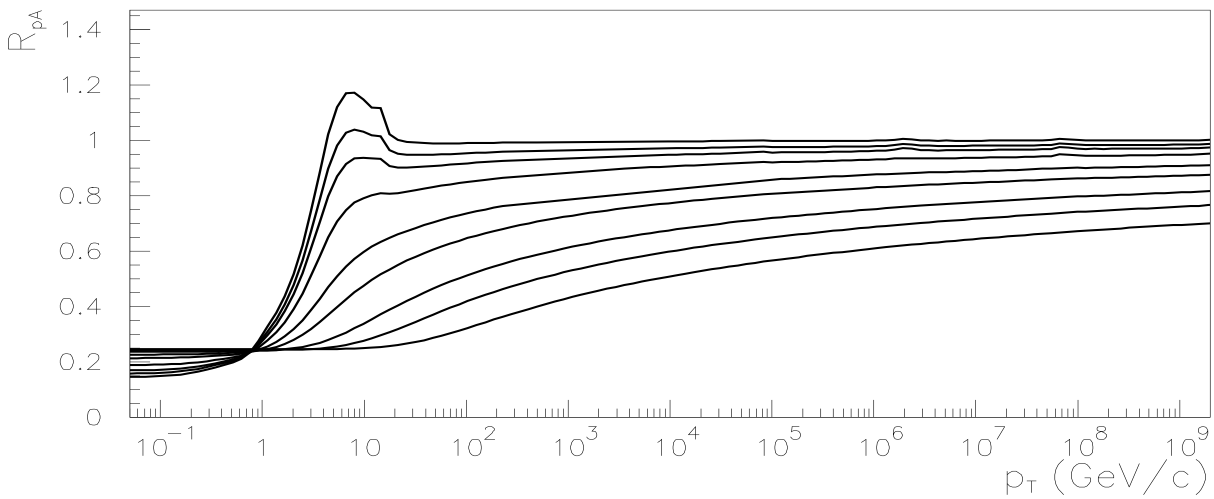


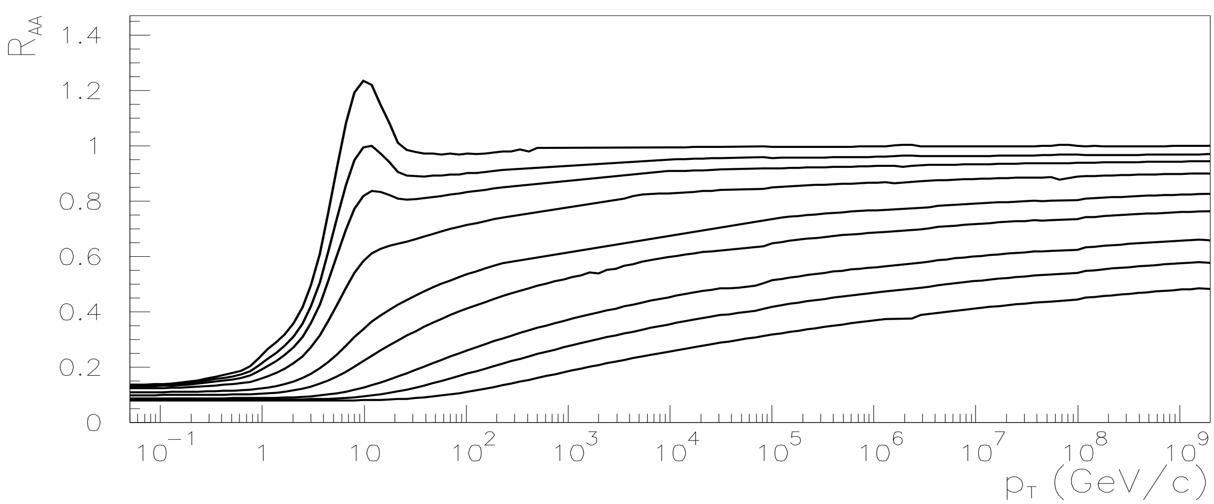
Quantum evolution a la
"super saturated"
non–local Gaussian of
Iancu,Itakura,McLerran

$$\mu^2 = \frac{4\pi}{g^2 N_c} k_t^2 \ln\left(1 + \left(\frac{Q_s}{k_t}\right)^{2\gamma}\right)$$

- Lesson: x not small enough at y=0–need MV–initial conditions at RHIC. May be small enough at y=3!

- Numerical solutinns of BK–equation with MV–initial conditions





Albacete,Armesto,Kovner,Salgado,Wiedemann

see also, Kharzeev,Kovchegov,Tuchin

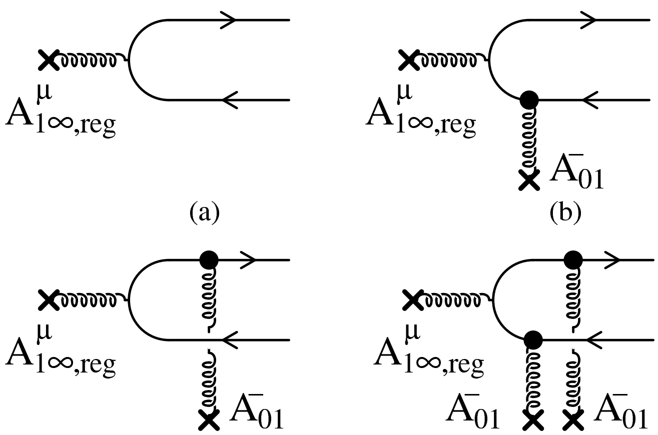
- Very important to understand running coupling BFKL effects!

Rummukainen & Weigert

Mueller & Triantafyllopolous

Quark production to all orders in pA

Blaizot,Gelis,RV

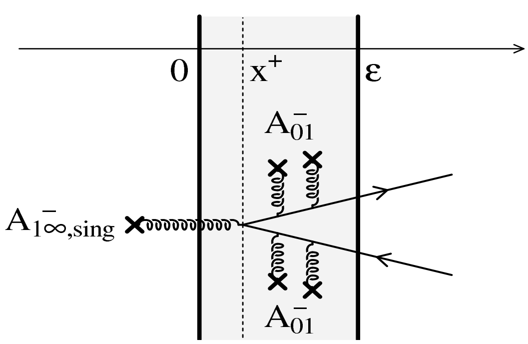


$A^{\mu}_{1\infty}$ is the gluon field

to $O(\rho_1 \rho_2^n)$ $n \to \infty$

- Computed both Feynman & retarded amplitudes– differ only by a phase.

- Again, the V–Wilson lines disappear–need contribution from pair scattering in nucleus

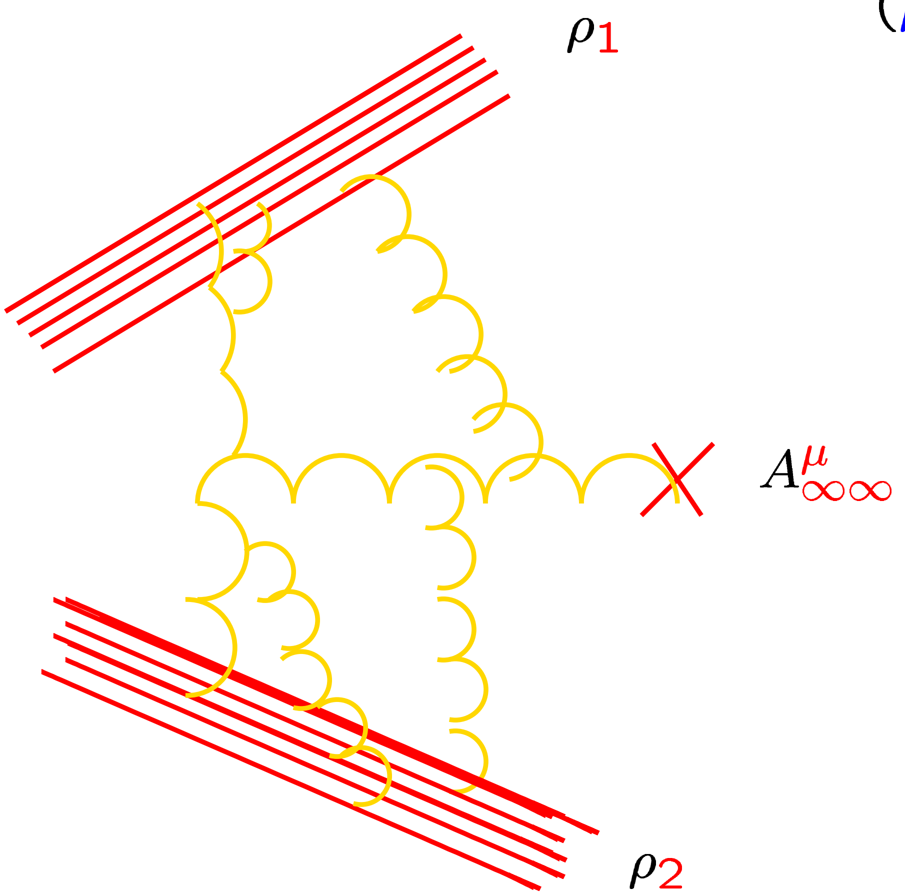


- Result for pair–production not k_t factorizable –not clear for single quark distributions.

See also Tuchin's talk for discussion of his work & work with Kharzeev

Gluon & Quark production in the dense/AA region

$(\rho_1/k_t^2, \rho_2/k_t^2 \sim 1)$



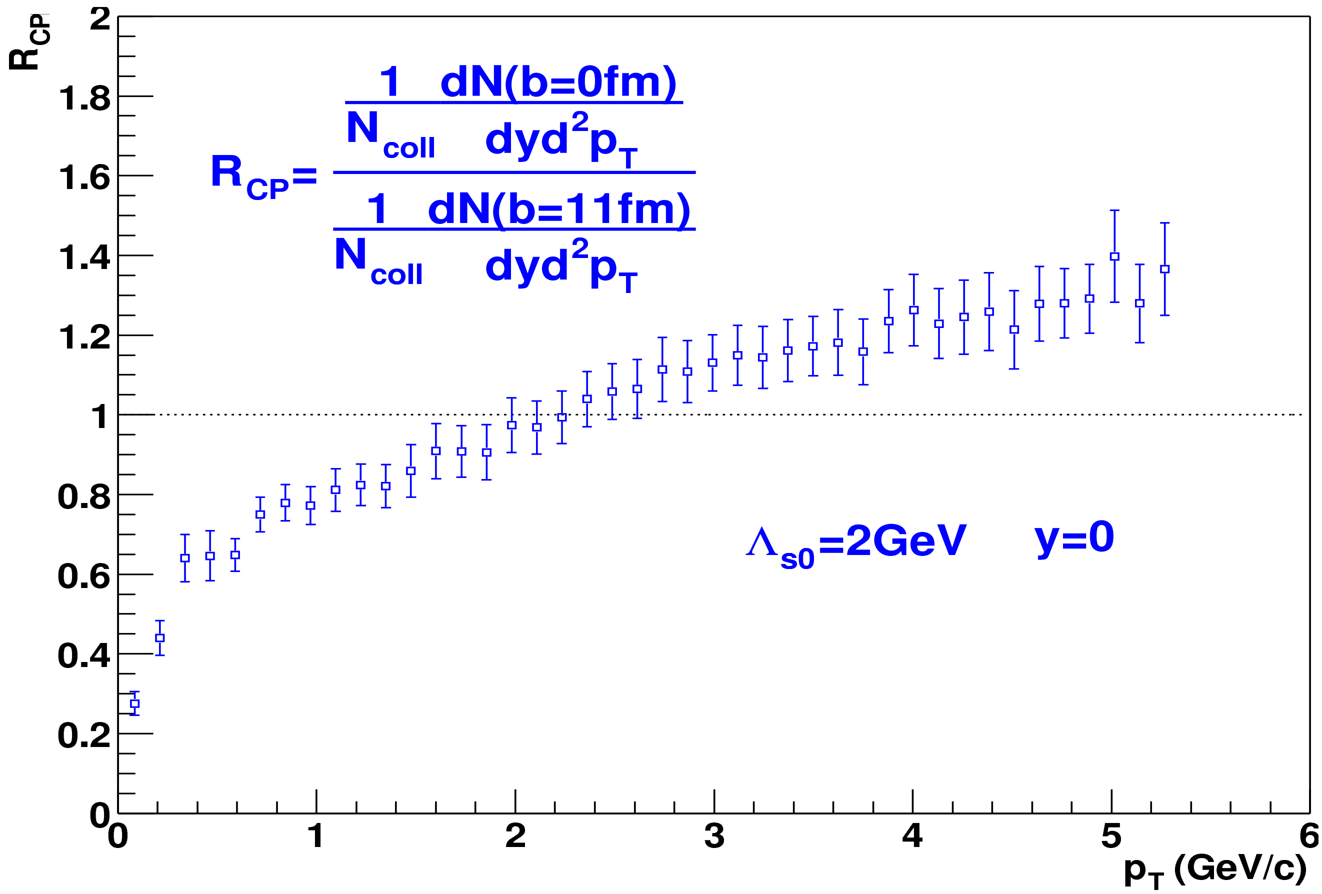
- Likely not $k_{\perp}$t factorizable–only solved numerically thus far

  Krasnitz,RV
  Krasnitz,Nara,RV
  Lappi

- Wave–fn evolution effects difficult to include –work of Rummukainen & Weigert promising...

- Classical evolution shows Cronin–hence re–scattering/energy–loss is strong at RHIC

Jalilian−Marian,Nara,RV



see also Baier,Kovner,Wiedemann

- Re−scattering conclusion also suggested by v_2 calculation of Krasnitz,Nara & RV

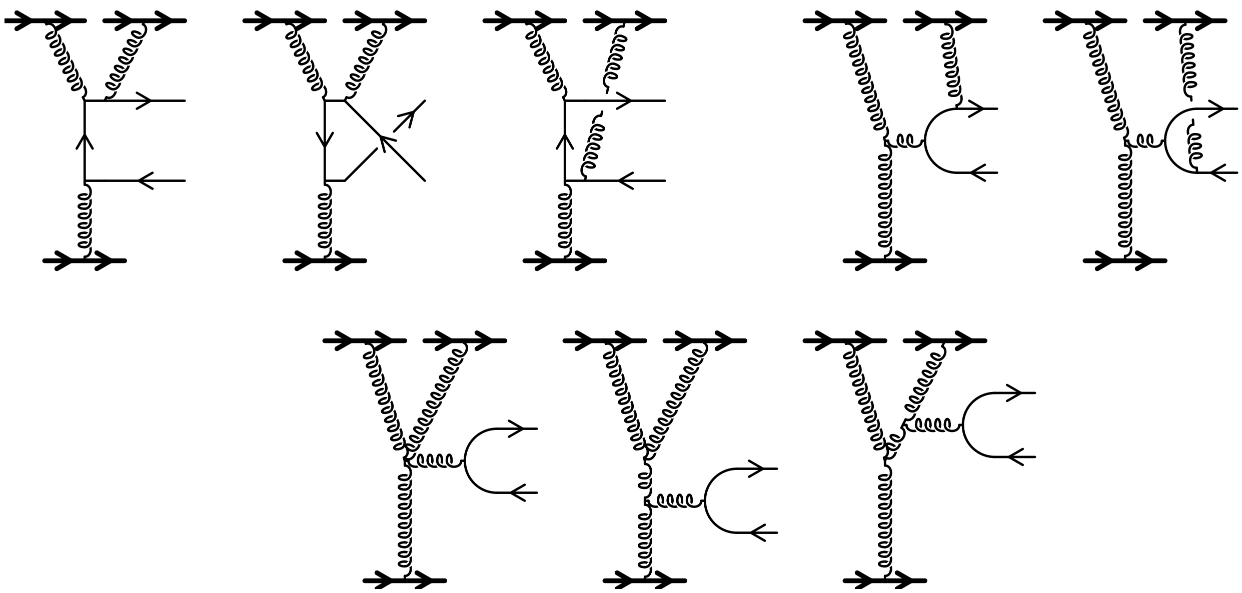
Study final state interactions beyond CGC−thermalization?

Baier,Mueller,Schiff,Son

Arnold, Lenaghan,Moore; Krasnitz, RV

Jeon,McLerran,RV,Weinstock
−work in progress

Quark Production in AA



Small sub−set of relevant diagrams...

- Tour de force numerical computation by Gelis, Kajantie, Lappi−in progress

Those cold, white nights in Helsinki...